The kinetostatic analysis of the shoulder girdle mechanism

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INTRODUCTION

In this paper a kinetostatic analysis is presented for an open kinematic chain, using a matrix formalism (Kovacs & Radulescu, 1992; Kovacs & Pommersheim, 1994). The study represents a rigid body approach which is able to calculate the joint moments and/or equilibration forces. The analysed kinematic chain is presented in Figure 1. Joint A represents the sternoclavicular joint, B the acromioclavicular joint, C is a point located on the spina scapulae close to the medial border, D the glenohumeral joint, and E the scapulothoracic gliding plane. Element 0, the basis of this kinematic chain is the sternum, element 1 is the clavicle, element 2 is the scapula and element 3 is the humerus. The other notations are introduced only to offer a global image of the human upper limb and will not be of importance for this paper.

This type of analysis implies the following input and output data:

- - Input data: the positions, the velocities and accelerations distribution for the mechanism, the resultant of external forces F_n and the resultant of external torques M_n; the index n refers to the n-th element of the considered kinematic chain;
- - Output data: the tensor of the dynamic reactions R_A, M_A for joint A and the dynamic equilibrium torque M or/and the dynamic equilibrium force F ; these entities, if applied in joint A, imply a well determined instantaneous movement and can be regarded as substituting the action of the external loading.

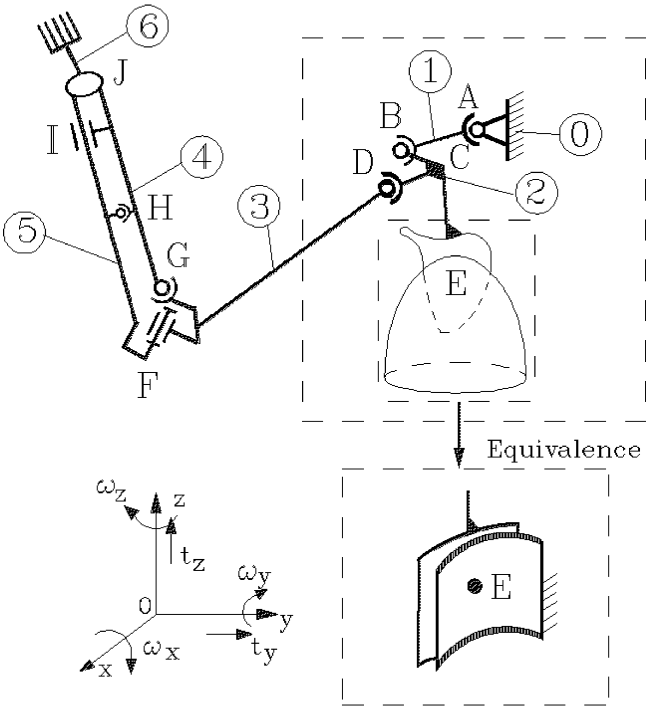

Figure 1-*The considered kinematic chain*

The main goal of this paper is to find the joint reaction forces and/or the joint moments over the shoulder girdle mechanism.

GENERAL PRESENTATION OF THE METHOD

For any actuating joint of the considered kinematic chain-in particular for joint A, next static equilibrium conditions could be written:

$$\vec{R}_A + \vec{F}_n = \vec{0}; \vec{M}_A + \vec{M}_{eA} + \vec{M}_n = \vec{0} \qquad (1)$$

For any actuating joint from the considered kinematic chain, in the hypothesis that the torsor of the external forces includes the equilibration force/torque, next conditions are true (example for joint D):

$$\vec{R}_D + \vec{F}_{eA} + \vec{F}_i = \vec{0}; \vec{M}_D + \vec{M}_i = \vec{0} \qquad (2)$$

For a concrete situation the equilibrium force or/and torque can be written as a matrix formalism, as presented in expression (3):

$$\begin{bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Az} \\ M_{Ax} \\ M_{Ay} \\ M_{Az} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} & J_{36} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} & J_{56} \\ J_{61} & J_{62} & J_{63} & J_{64} & J_{65} & J_{66} \end{bmatrix} \cdot \begin{bmatrix} F_{nx} \\ F_{ny} \\ F_{nz} \\ M_{nx} \\ M_{ny} \\ M_{nz} \end{bmatrix} \qquad (3)$$

where J_{ij} terms are the terms of the Jacobean matrix attached to the system. If developing the first line of this matrix, next relation is obtained:

$$R_{Ax} = J_{11}F_{nx} + J_{12}F_{ny} + J_{13}F_{nz} + \dots$$
$$\dots J_{14}M_{nx} + J_{15}M_{ny} + J_{16}M_{nz} =$$
$$\frac{\partial R_{Ax}}{\partial F_{nx}} \cdot F_{nx} + \frac{\partial R_{Ax}}{\partial F_{ny}} \cdot F_{ny} + \frac{\partial R_{Ax}}{\partial F_{nz}} \cdot F_{nz} + \dots \quad (4)$$
$$\dots \frac{\partial R_{Ax}}{\partial M_{nx}} \cdot M_{nx} + \frac{\partial R_{Ax}}{\partial M_{ny}} \cdot M_{ny} + \frac{\partial R_{Ax}}{\partial M_{nz}} \cdot M_{nz}$$

It can be observed that equation (4) represents the expression of developing in Taylor series (only the linear terms) of $R_{Ax}(F_{nx}, F_{ny}, F_{nz}, M_{nx}, M_{ny}, M_{nz})$. Next the situation for joints presenting one degree of freedom presenting rotational movement-R and translational movement-T is presented.

JOINTS WITH ONE DEGREE OF FREEDOM (ROTATIONAL-R OR TRANSLATION-T)

This section will show the calculus relations for calculating the reaction force and the moment in a joint presenting one degree of freedom. The joints of the considered kinematic chain (Figure 1) will be considered as "collections" of such simple joints. Considering the situation presented in Figure 2, and in the hypothesis of non-friction, for the R joint, next equilibrium conditions can be written:

$$\vec{R}_A + \vec{F}_n = \vec{0} \quad (5)$$

$$\vec{M}_A + \vec{M}_{eA} + \vec{M}_n + \vec{h}_A \times \vec{R}_A + \vec{d}_n \times \vec{F}_n = 0 \quad (6)$$

where:

- $\vec{F}_n = \left[F_{nx} + F_{ny} + F_{nz}\right]^T$ is the resultant of external forces;
- $\vec{M}_n = \left[M_{nx} + M_{ny} + M_{nz}\right]^T$ is the resultant torque of the external forces;
- $\vec{d}_n = \left[d_{nx} + d_{ny} + d_{nz}\right]^T$ is the perpendicular on the direction of $\vec{F}_n$ from the origin;
- $\vec{R}_A = \left[R_{Ax} + R_{Ay} + R_{Az}\right]^T$ is the reaction force in the joint;
- $\vec{h}_A = \left[h_{Ax} + h_{Ay} + h_{Az}\right]^T$ is the perpendicular on the direction of $\vec{R}_A$ from the origin;
- $\vec{M}_A = \left[M_{Ax} + M_{Ay} + 0\right]^T$ is the reaction torque in the kinematic joint, being perpendicularly directed on Oz axis;
- $\vec{M}_{eA} = \left[0 + 0 + M_{eA}\right]^T$ is the equilibrium torque in the direction of the Oz axis, having as effect the movement of n-th element reported to the n-1-th element.

The problem is to find the terms J_{ij} (i,j=1,2,...,6) of the Jacobean matrix. For this, the vectorial equilibrium relations will be projected on the axiss of the reference system. For forces we have:

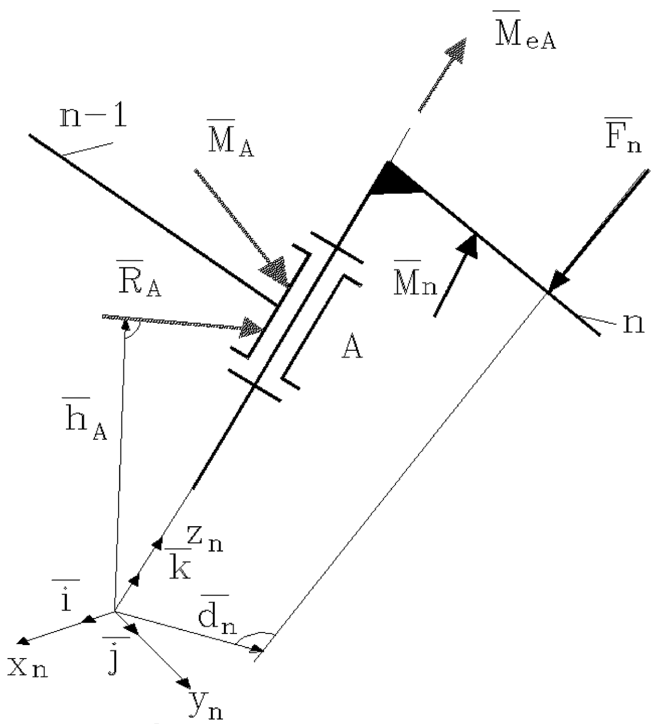

Figure 2-*R kinematic joint*

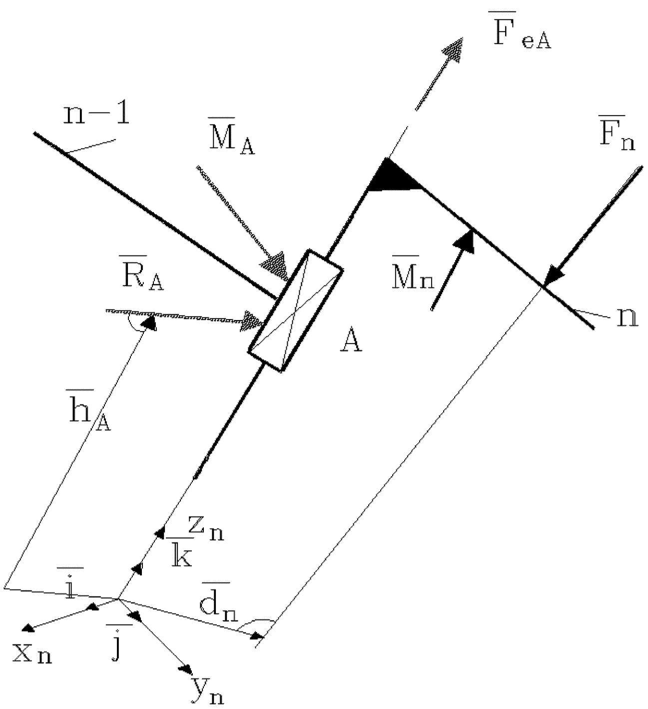

Figure 3-*Kinematic joint T*

$$\begin{bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Az} \end{bmatrix} = -\begin{bmatrix} F_{nx} \\ F_{ny} \\ F_{nz} \end{bmatrix} \qquad (7)$$

the corresponding J_{ij} terms being presented in the relations (8) - (10).

$$J_{11} = \frac{\partial R_{Ax}}{\partial F_{nx}} = -1, J_{12} = J_{13} = J_{14} = J_{15} = J_{16} = 0; \qquad (8)$$

$$J_{22} = \frac{\partial R_{Ay}}{\partial F_{ny}} = -1, J_{21} = J_{23} = J_{24} = J_{25} = J_{26} = 0; \qquad (9)$$

$$J_{33} = \frac{\partial R_{Az}}{\partial F_{nz}} = -1, J_{31} = J_{32} = J_{34} = J_{35} = J_{36} = 0. \qquad (10)$$

Proceeding in an analogous way, for torques, after calculus, the other J_{ij} will be found, the Jacobean matrix having the expression:

$$J^{rot} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -(h_{Az} - d_{nz}) & h_{Ay} - d_{ny} & -1 & 0 & 0 \\ h_{Az} - d_{nz} & 0 & -(h_{Ax} - d_{nx}) & 0 & -1 & 0 \\ -(h_{Ay} - d_{ny}) & h_{Ax} - d_{nx} & 0 & 0 & 0 & -1 \end{bmatrix} \qquad (11)$$

In this way, for this situation, the matrix equation characterising from kinetostatic point of view the considered system is:

$$\begin{bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Az} \\ M_{Ax} \\ M_{Ay} \\ M_{eA} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -(h_{Az} - d_{nz}) & h_{Ay} - d_{ny} & -1 & 0 & 0 \\ h_{Az} - d_{nz} & 0 & -(h_{Ax} - d_{nx}) & 0 & -1 & 0 \\ -(h_{Ay} - d_{ny}) & h_{Ax} - d_{nx} & 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} F_{nx} \\ F_{ny} \\ F_{nz} \\ M_{nx} \\ M_{ny} \\ M_{nz} \end{bmatrix} \qquad (12)$$

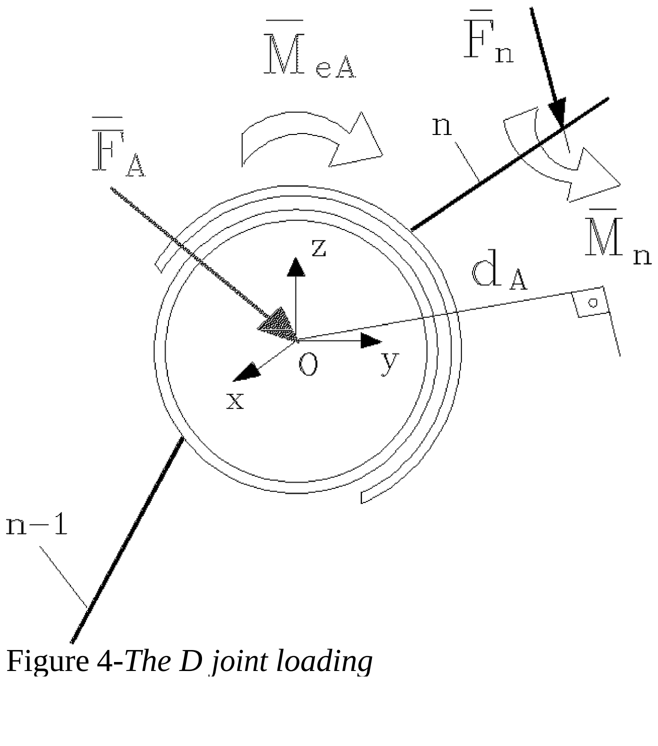

Figure 4-*The D joint loading*

The significance of the all entities from relation (12) were formerly presented. We underline the fact that in the present case it was analysed the situation of rotation around Oz axis. When rotation after Ox or Oy axis is done, the characteristic equations, like relation (3), are obtained in an analogue way.

Considering Figure 3, in the hypothesis of no friction, for the case of a T joint, next equilibrium relations can be written:

$$\vec{R}_A + \vec{F}_{eA} + \vec{F}_n = \vec{0} \qquad (13)$$

$$\vec{M}_A + \vec{M}_n + h_A \times \vec{R}_A + d_n \times \vec{F}_n = \vec{0} \qquad (14)$$

where:

$\vec{R}_A = \left[R_{Ax} + R_{Ay} + 0 \right]^T$

- is the resultant and it action in a direction perpendicular on Oz axis;

$\vec{h}_A = \left[0 + 0 + h_{Az} \right]^T$

- is parallel with Oz axis;

$\vec{M}_A = \left[M_{Ax} + M_{Ay} + M_{Az} \right]^T$

- is the reaction torque;

$\vec{F}_{eA} = \left[0 + 0 + F_{eA} \right]^T$

- is the equilibration force that acts in the direction

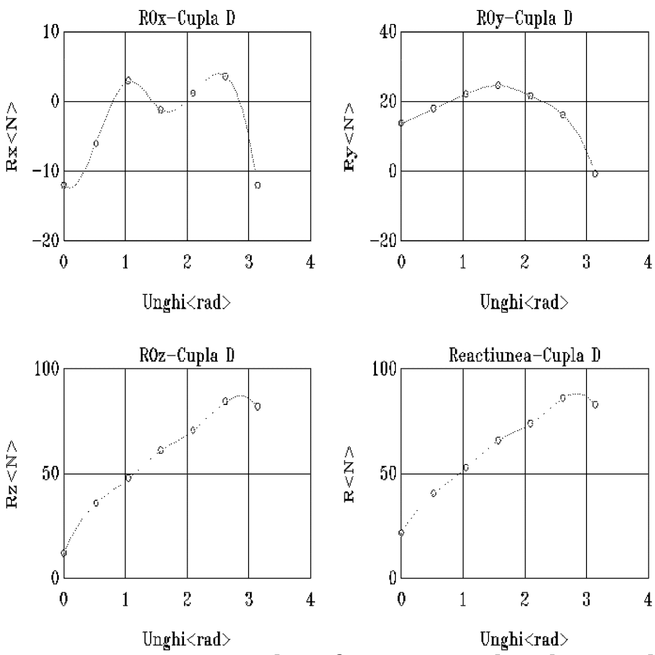

Figure 5-*Reaction values for D joint-glenohumeral joint (Rx, Ry, Rz- reaction force in projections on Ox, Oy and Oz axis; R-reaction force)*

of the Oz axis, allowing the relative movement of n-th element reported to the n+1-th element.

The matrix equation that characterises the system from a from kinetostatic point of view is:

$$\begin{bmatrix} R_{Ax} \\ R_{Ay} \\ F_{eA} \\ M_{Ax} \\ M_{Ay} \\ M_{Az} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -(h_{Az} - d_{nz}) & -d_{ny} & -1 & 0 & 0 \\ h_{Az} - d_{nz} & 0 & d_{nx} & 0 & -1 & 0 \\ d_{ny} & -d_{nx} & 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} F_{nx} \\ F_{ny} \\ F_{nz} \\ M_{nx} \\ M_{ny} \\ M_{nx} \end{bmatrix} \quad (15)$$

SOME OF THE OBTAINED RESULTS FOR THE SHOULDER GIRDLE MECHANISM

In this paragraph the following values are considered as input data: the positions, the velocities and accelerations distribution for the mechanism, the resultant of external forces F_n and the resultant of external torques M_n ; the index n refers to the n-th element of the considered kinematic chain. The model will not consider the inertial forces and torques because they present, for most of the daily normal movements, small values. The resultant of the external forces and external torques were introduced as presented in [1], [2], [3], [6] for the unloaded abduction movement.

The D joint presents an important functional role in the considered (bio)kinematic chain (Figure 4) It can be observed that it presents only reaction forces, the reaction torques being zero.

It is now possible to find the relation that allows to find all six reaction components:

$$\begin{bmatrix} R_{Dx} \\ R_{Dy} \\ R_{Dz} \\ M_{exD} \\ M_{eyD} \\ M_{ezD} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -(h_{Dz} - d_{3z}) & h_{Dy} - d_y & -1 & 0 & 0 \\ h_{Dz} - d_{3z} & 0 & -(h_{Dx} - d_{3x}) & 0 & -1 & 0 \\ -(h_{Dy} - d_{3y}) & h_{Dx} - d_{3x} & 0 & 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} F_{3x} \\ F_{3y} \\ F_{3z} \\ M_{3x} \\ M_{3y} \\ M_{3z} \end{bmatrix} \quad (16)$$

Figure 5 presents the reaction forces in D joint and Figure 6 shows the values for the equilibration torque on the humerus (the 3rd element). The curves are obtained using B-spline interpolations. Figure 7 presents the value of the reaction forces in A (sternoclavicular), B (acromioclavicular) and E (scapulothoracic gliding plane) joints. Figure 8 presents the equilibration forces and torques.

CONCLUSIONS

The present analysis presents few advantages (marked +) and some weak points (marked -):

- + It can be easily used for simulating non physiologic loading over the shoulder girdle and the upper limb, eliminating sophisticated research; could be possible to have a general idea concerning the magnitude of solicitations, for example in the case of an accident;
- + The numerical analysis can be done on usual PCs, using common software packages;
- - The obtained data for A and B joints do not agree with biological information, the main reason being that the external forces were locally implemented and not dividing the muscular action in many line action forces [1];

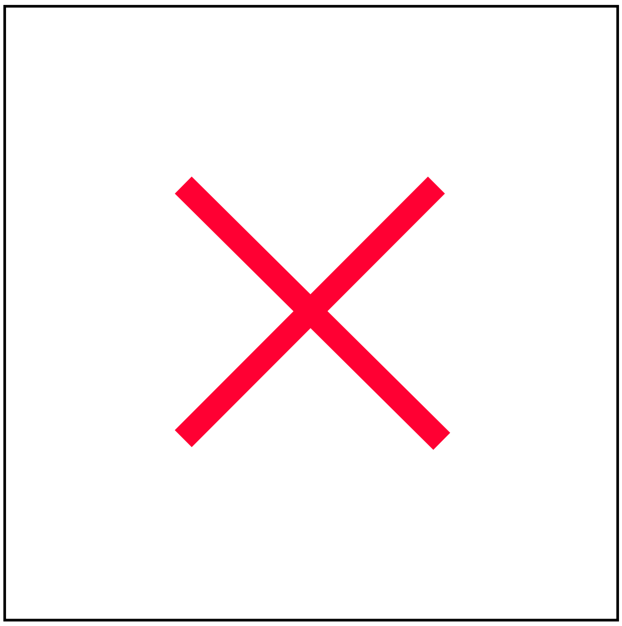

Figure 6-*Equilibrium torque values for 3rd element- the humerus (projections on the co-ordinate system axiss and the magnitude)*

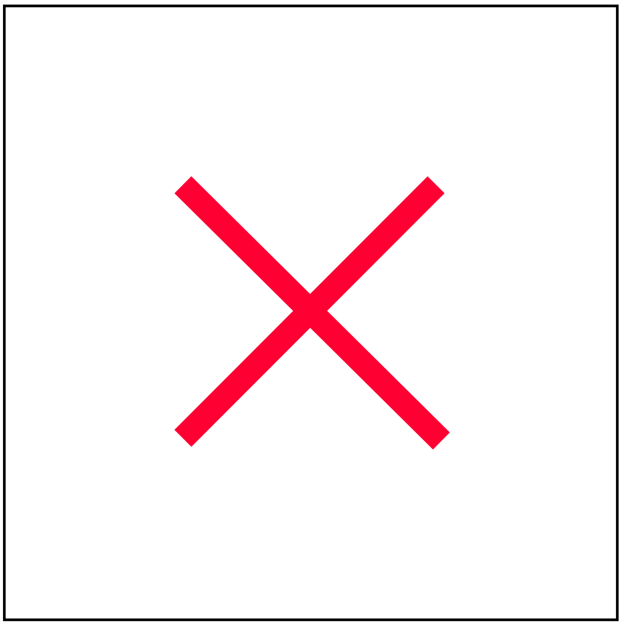

Figure 7 -*The reaction values for B- acromioclavicular, A- sternoclavicular, E-scapulothoracic gliding plane joints; B joint-marked "o" , A joint-marked" *"; E joint- marked "+"*

- - The inertia forces and torques were not taken into consideration; for slow movement this represents an acceptable hypothesis;
- - The elasticity of the biological structures was not considered.

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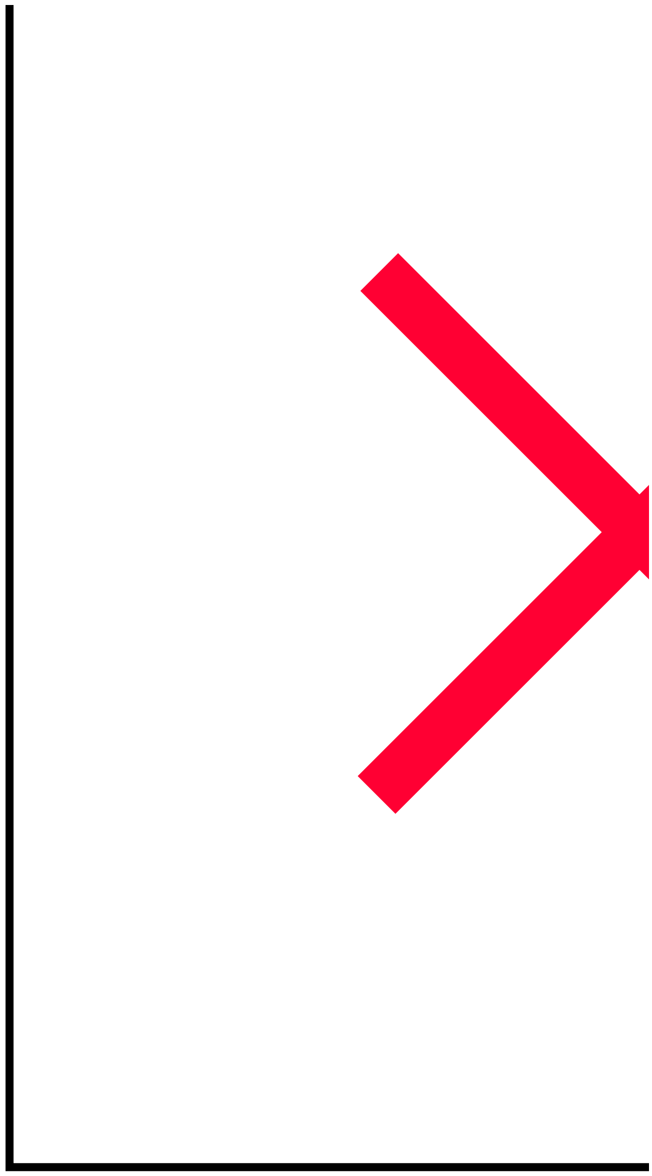

Figure 8 - *Equilibration torques and forces; Element 1(clavicle) marked "o", equilibration torque; Element 2 (scapula), equilibration torque; Element 2 (scapula) marked" *", equilibration force (x50)*