

# Musculo-skeletal modelling of the shoulder

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This document is also available online at:  
<http://www.internationalshouldergroup/>

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*Introduction*

*Equations of motion*

*Inverse-dynamic...*

*Examples of inverse...*

*Muscle dynamics*

*Forward dynamics*

*Examples of forward...*

*Summary*

*Title Page*

◀◀ ▶▶

◀ ▶

*Page 1 of 42*

*Go Back*

*Full Screen*

*Close*

*Quit*

# Contents

- Introduction
  - goals
  - requirements
- Equations of motion
  - Newton-Euler, Lagrange, TMT
- Inverse-dynamic modelling
  - input/output
  - the load-sharing problem
  - constraining the optimisation
- Examples
  - effect of constraints
  - moment balance
  - joint forces

*Introduction*

*Equations of motion*

*Inverse-dynamic...*

*Examples of inverse...*

*Muscle dynamics*

*Forward dynamics*

*Examples of forward...*

*Summary*

*Title Page*



*Page 2 of 42*

*Go Back*

*Full Screen*

*Close*

*Quit*

- Muscle dynamics
  - muscle dynamics in the optimisation
  - muscle model
- Inverse-dynamics summary
- Forward dynamic modelling
  - input/output
  - advantages of forward dynamics
  - calculating the input
  - integration methods
- Examples
  - prosthesis placement
  - computer assisted surgery
- Summary

*Introduction*

*Equations of motion*

*Inverse-dynamic...*

*Examples of inverse...*

*Muscle dynamics*

*Forward dynamics*

*Examples of forward...*

*Summary*

*Title Page*



*Page 3 of 42*

*Go Back*

*Full Screen*

*Close*

*Quit*

# 1. Introduction

## 1.1. Goals

Biomechanical models should allow:

- Analysis of clinical problems
  - diagnosis of disorders
  - improvement of current treatments
  - development of new treatments
- Insight into human function
  - muscle function
  - coordination
  - energy usage
  - muscle and joint forces
- Computer-assisted surgery
  - optimisation of treatment for specific patient

|                               |
|-------------------------------|
| <i>Introduction</i>           |
| <i>Equations of motion</i>    |
| <i>Inverse-dynamic...</i>     |
| <i>Examples of inverse...</i> |
| <i>Muscle dynamics</i>        |
| <i>Forward dynamics</i>       |
| <i>Examples of forward...</i> |
| <i>Summary</i>                |

Title Page

◀◀ ▶▶

◀ ▶

Page 4 of 42

Go Back

Full Screen

Close

Quit

## 1.2. Requirements

To achieve these goals, we need:

- Large-scale, 3D, comprehensive model
  - necessary to answer specific clinical questions
  - essential if validation is to be achieved
  
- Interpretable results
  - can we relate model output to patient function?
  
- Fast model, user-friendly interface
  - for use in the clinical setting

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 5 of 42

Go Back

Full Screen

Close

Quit

### 1.3. Specific requirements of a shoulder model

#### Segments

- thorax, scapula, clavicle, humerus, ulna, radius, hand

#### Degrees of freedom

|                                |           |
|--------------------------------|-----------|
| thorax                         | 6         |
| sterno-clavicular joint        | 3         |
| acromio-clavicular joint       | 3         |
| gleno-humeral joint            | 3         |
| humero-ulnar                   | 1         |
| radio-ulnar                    | 1         |
| wrist                          | 3         |
| scapulo-thoracic gliding plane | -2        |
| conoid ligament                | -1        |
| <b>Total</b>                   | <b>17</b> |

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 6 of 42

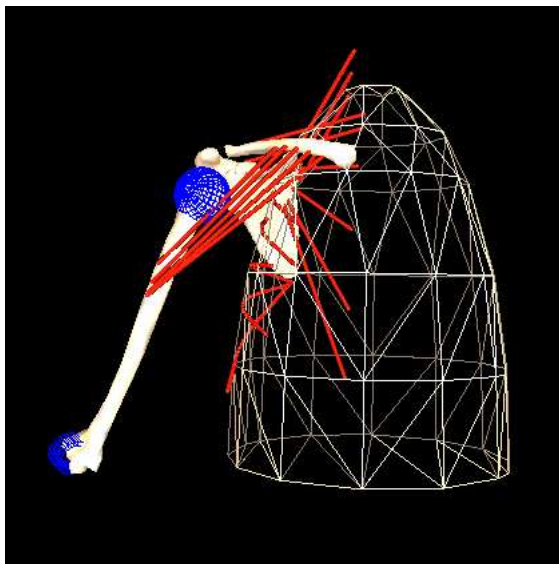
Go Back

Full Screen

Close

Quit

## 1.4. Example of a shoulder model



SIMM representation of the Delft Shoulder Model

Full description in [van der Helm \(1994\)](#)

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 7 of 42

Go Back

Full Screen

Close

Quit

## 2. Equations of motion

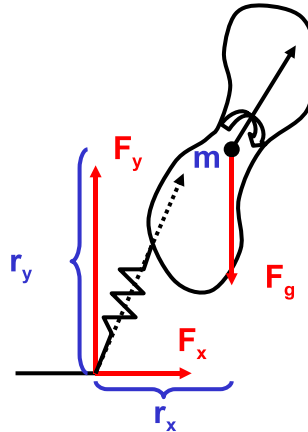
### 2.1. Newton-Euler method

Free body diagram in 2D:

$$F_x - m\ddot{x}_x = 0 \quad (1)$$

$$F_y - m\ddot{x}_y - mg = 0 \quad (2)$$

$$F_x r_y - F_y r_x - I\ddot{\theta} = 0 \quad (3)$$



This method results in a large set of algebraic equations and is cumbersome and time-consuming for multi-body systems.

Discussed in Zajac et al. (2002).

Title Page



Page 8 of 42

Go Back

Full Screen

Close

Quit

## 2.2. Lagrange method

The Lagrange method allows us to express the equations of motion in terms of generalised coordinates (degrees of freedom), using a minimum number of equations.

The system Lagrangian,  $L$ , is given by

$$L = T - V \quad (4)$$

where  $T$  is the kinetic energy, and  $V$  the potential energy.

Equations of motion are:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \left( \frac{\partial L}{\partial q_i} \right) = M_i \quad (5)$$

where  $M_i$  are the generalised moments.

This method results in a minimal set of ordinary differential equations (ODE) which can be numerically integrated.

### 2.3. TMT method

TMT is a combination of Newton-Euler and Lagrange.

Start with the Newton-Euler equation:

$$\sum (f_i - M_{ij} \cdot \ddot{x}_j) = 0 \quad (6)$$

Transfer function,  $T_i$ , describes output vector,  $x_i$ , in terms of generalised coordinates,  $q_j$ :

$$x_i = T_i(q_j) \quad (7)$$

Differentiation of (7) w.r.t. time gives:

$$\dot{x}_i = \frac{\partial T_i}{\partial q_j} \cdot \dot{q}_j = T_{i,j} \cdot \dot{q}_j \quad (8)$$

and again:

$$\ddot{x}_i = \frac{\partial T_{i,j}}{\partial q_k} \cdot \dot{q}_j \cdot \dot{q}_k + T_{i,j} \cdot \ddot{q}_j = T_{i,jk} \cdot \dot{q}_j \cdot \dot{q}_k + T_{i,j} \cdot \ddot{q}_j \quad (9)$$

Substitution of (9) into (6) gives:

$$\sum f_i - M_{ij} (T_{i,jk} \cdot \dot{q}_j \cdot \dot{q}_k + T_{i,j} \cdot \ddot{q}_j) = 0 \quad (10)$$

Premultiplying by  $T_{i,j}^T$  and rearranging gives:

$$T_{i,j}^T \cdot M_{ij} \cdot T_{i,j} \cdot \ddot{q}_j = T_{i,j}^T \cdot \sum f_i - T_{i,j}^T \cdot M_{ij} \cdot T_{i,jk} \cdot \dot{q}_j \cdot \dot{q}_k \quad (11)$$

Simplifying,

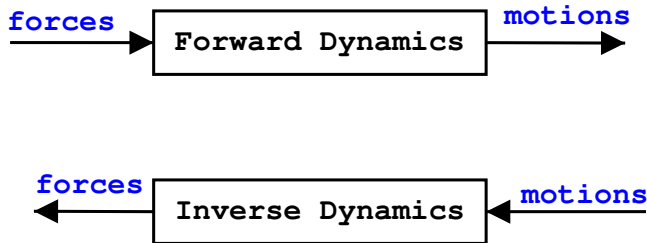
$$\ddot{q} = f(t, \dot{q}) \quad (12)$$

which are the equations of motion in terms of an ODE.

- simple derivation of equations of motion
- number of equations = number of DoF
- suitable for numerical methods (see 6.4)

Described by [van Soest et al. \(1992\)](#) and [Jonker \(1984\)](#).

## 2.4. Inverse or forward dynamics?



Inverse dynamics:

- + efficient optimisation
- assumptions about kinematics

Forward dynamics:

- + no assumptions about kinematics
- optimisation very expensive

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 12 of 42

Go Back

Full Screen

Close

Quit

### 3. Inverse-dynamic modelling

#### 3.1. Input/Output



**input**

recorded motions  
external loads

**output**

muscle lengths  
moment arms  
net moments  
muscle forces  
joint forces  
muscle powers

- Introduction
- Equations of motion
- Inverse-dynamic...**
- Examples of inverse...
- Muscle dynamics
- Forward dynamics
- Examples of forward...
- Summary

Title Page

◀ ▶

◀ ▶

Page 13 of 42

Go Back

Full Screen

Close

Quit

## 3.2. Model input

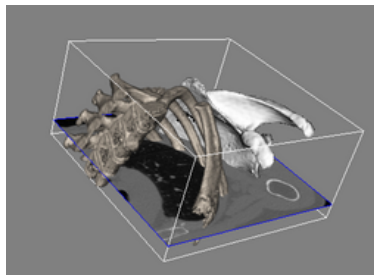
Input motion can be kinematically incompatible with model geometry due to closed-chain mechanism.

Ideally,

⇒ scale model to subject geometry

Best solution but difficult:

- clavicle length
- scapula dimensions
- conoid length
- thorax dimensions



Work in progress! [click here to see automatic segmentation](#)  
 More information at: <http://visualisation.tudelft.nl/>

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 14 of 42

Go Back

Full Screen

Close

Quit

Workaround:

⇒ adjust input angles to satisfy constraints

Minimise  $J$ , where

$$J = \sum_{i=1}^6 (\theta_{i_{measured}} - \theta_{i_{optimised}})^2 \quad (13)$$

where  $\theta_{1-6}$  are the angles of the clavicle and scapula

subject to the following constraints:

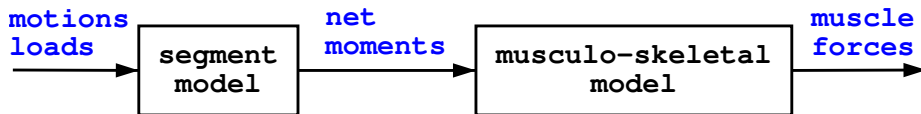
- scapula remains on thorax
- conoid length constant

[Title Page](#)


Page 15 of 42

[Go Back](#)
[Full Screen](#)
[Close](#)
[Quit](#)

### 3.3. Stages of inverse-dynamic modelling



length  
 diameter  
 mass  
 centre of mass  
 moment of inertia

joint rotation centres  
 muscle attachments  
 muscle wrapping  
 PCSA

- Introduction
- Equations of motion
- Inverse-dynamic...**
- Examples of inverse...
- Muscle dynamics
- Forward dynamics
- Examples of forward...
- Summary

Title Page

◀ ▶

◀ ▶

Page 16 of 42

Go Back

Full Screen

Close

Quit

### 3.4. The load-sharing problem

Many more muscles than degrees of freedom

⇒ System is indeterminate (the 'load-sharing problem')

⇒ **Optimisation:** minimise some performance criterion,  $J$

where  $J$  is one of the following:

- sum of squared muscle forces
- sum of squared muscle stresses
- maximum muscle stress
- fatigue
- energy consumption
- others...

See [Tsirakos et al. \(1997\)](#) for a discussion of different cost functions.

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 17 of 42

Go Back

Full Screen

Close

Quit

### 3.5. Constraining the optimisation

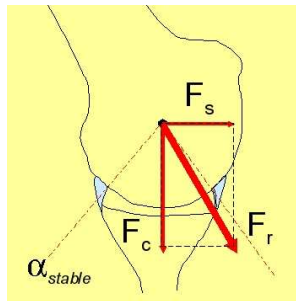
In addition to the moment balance, there are four other constraints on the optimisation:

The first is obvious:

- muscle forces must be non-negative

The next applies specifically to the gleno-humeral joint:

- GH joint stability must be ensured

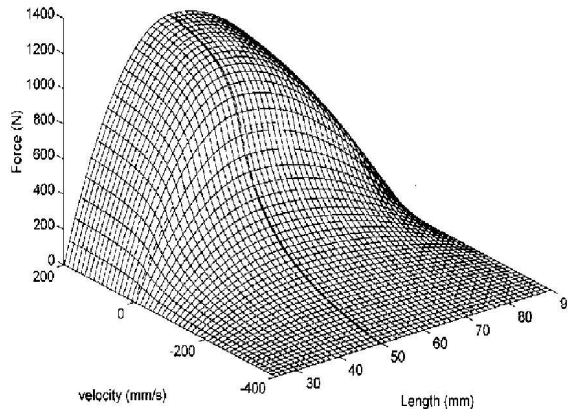


The next applies to the scapulo-thoracic gliding plane:

- contact force between thorax and scapula must be negative (compression)

Finally,

- muscles can only generate force within a certain range of lengths and velocities



Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 19 of 42

Go Back

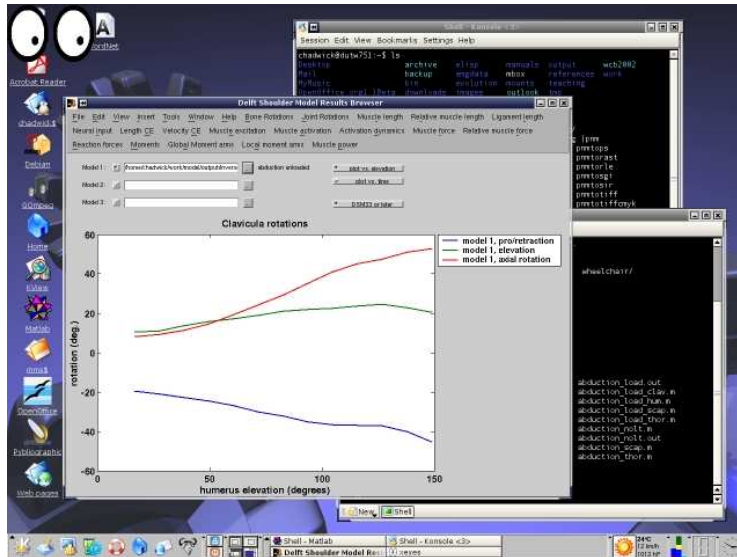
Full Screen

Close

Quit

## 4. Examples of inverse dynamics

### 4.1. Matlab results viewer



⇒ (5)

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 20 of 42

Go Back

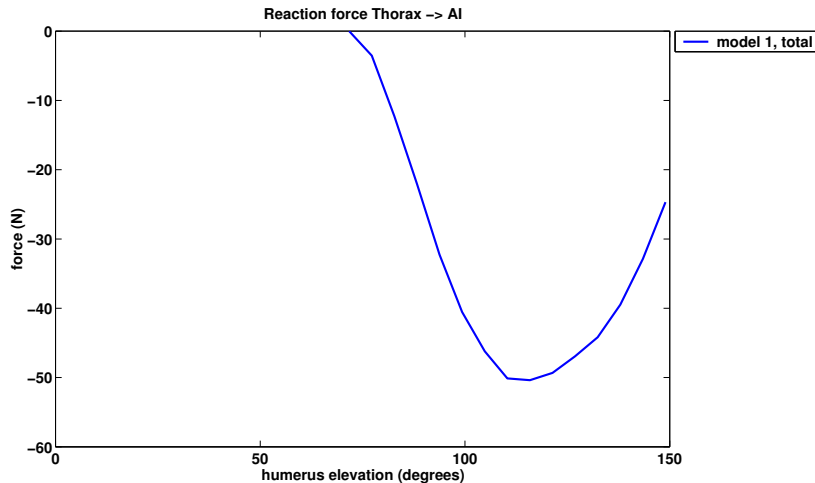
Full Screen

Close

Quit

## 4.2. Effects of constraints: scapulo-thoracic contact

Anteflexion of the humerus:



Force between AI and thorax is zero at beginning of movement, showing that the scapulo-thoracic constraint is active to keep the scapula against the thorax.

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 21 of 42

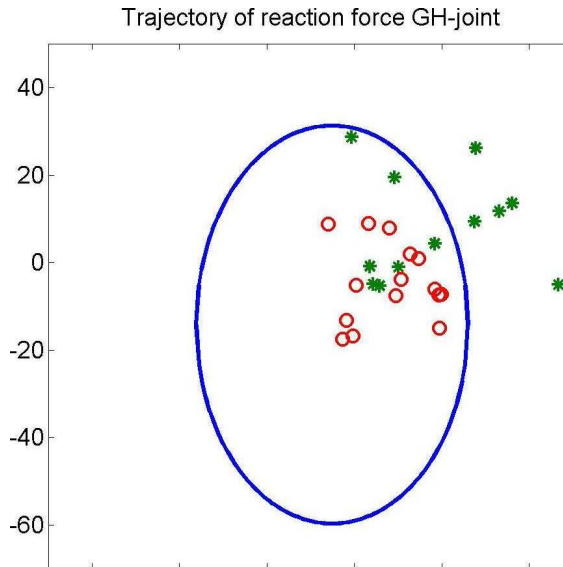
Go Back

Full Screen

Close

Quit

### 4.3. Effects of constraints: gleno-humeral stability



position of JRF in glenoid: green without rotator cuff

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 22 of 42

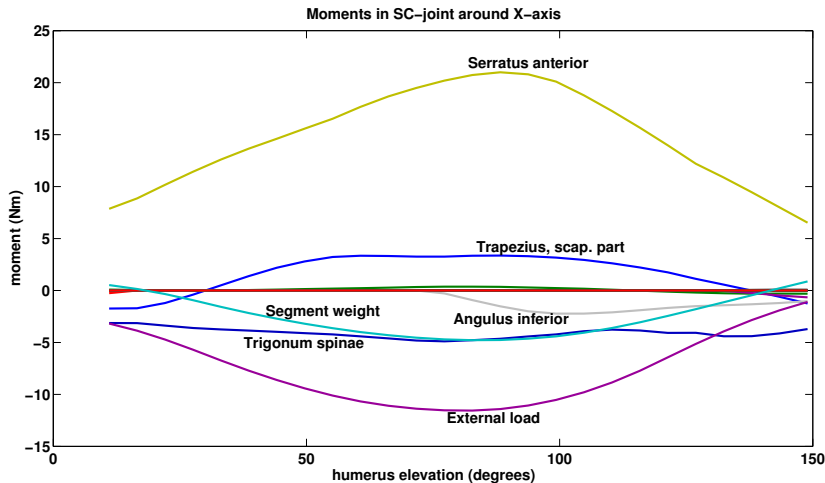
Go Back

Full Screen

Close

Quit

## 4.4. Moment balance: SCx during loaded anteflexion (2kg)



moments in the sagittal plane

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 23 of 42

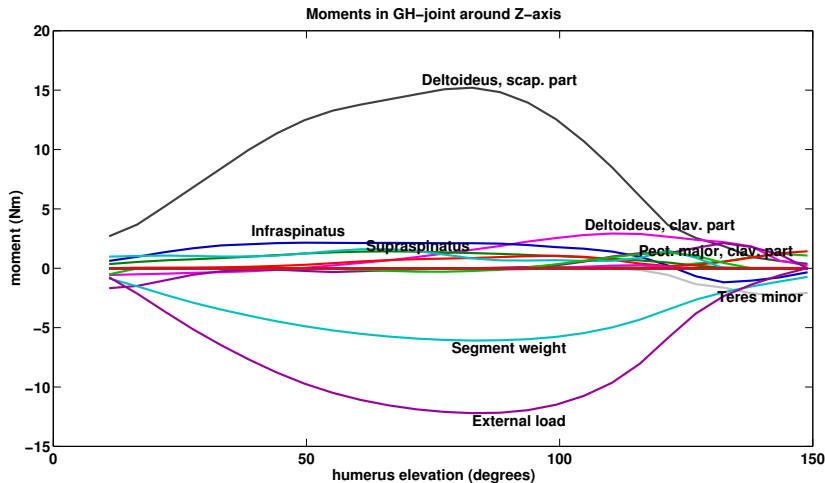
Go Back

Full Screen

Close

Quit

## 4.5. Moment balance: GH during loaded abduction (2kg)



moments in the frontal plane

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 24 of 42

Go Back

Full Screen

Close

Quit

- Introduction
- Equations of motion
- Inverse-dynamic...
- Examples of inverse...**
- Muscle dynamics
- Forward dynamics
- Examples of forward...
- Summary

## 4.6. Gleno-humeral joint forces

**Question:** What is the loading on the gleno-humeral joint during wheelchair propulsion?

**Context:** Motions are known (measured from volunteers), muscle and joint forces only required.

**Tool:** Inverse-dynamic model.

**Application:** Understanding shoulder pain in wheelchair users.

Veeger et al. (2002)

Title Page

◀◀ ▶▶

◀ ▶

Page 25 of 42

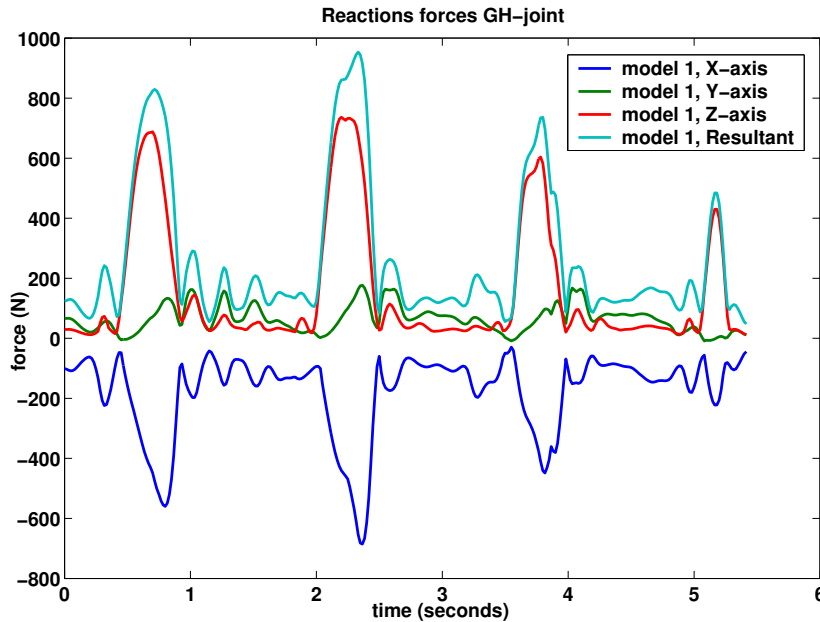
Go Back

Full Screen

Close

Quit

## 4.7. Gleno-humeral joint forces during wheelchair propulsion



Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 26 of 42

Go Back

Full Screen

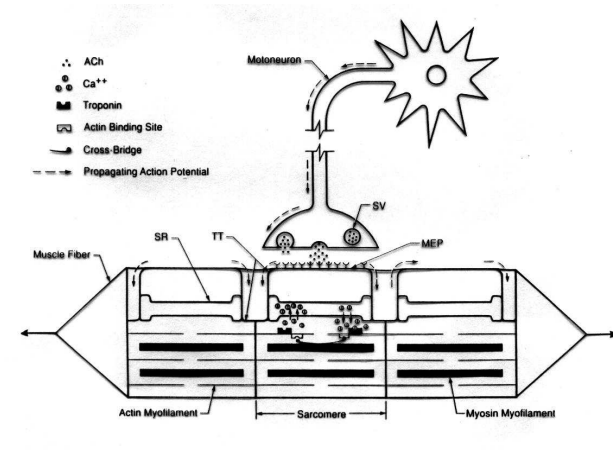
Close

Quit

## 5. Muscle dynamics

But

- muscle force cannot change instantaneously



⇒ include muscle dynamics in optimisation

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 27 of 42

Go Back

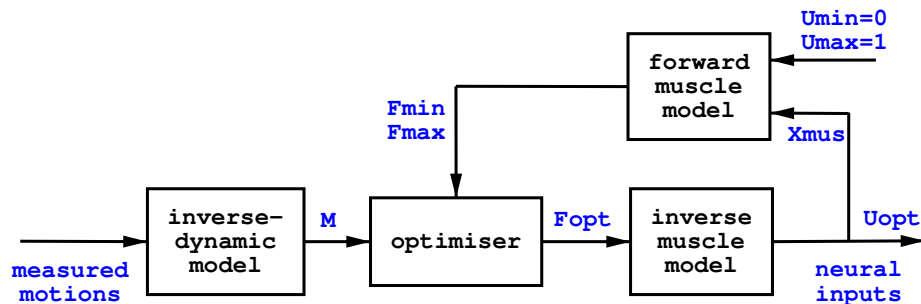
Full Screen

Close

Quit

## 5.1. Inverse-/Forward-Dynamic Optimisation (IFDO)

The frames of the motion are no longer time-independent with the inclusion of muscle dynamics, so the scheme becomes a little more complicated:

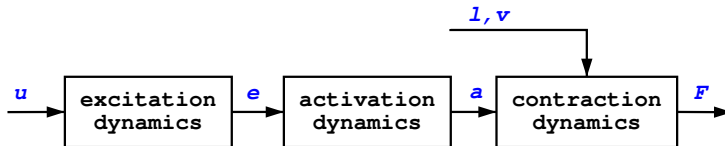


$F_{min}$  and  $F_{max}$  are calculated by integration of the muscle states to the next time-step, and ensure physiologically feasible bounds for the optimisation.

Happee and Van der Helm (1995)

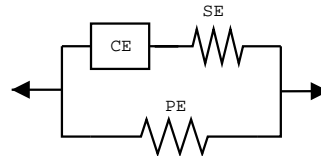
## 5.2. Muscle model

Hill-type muscle model:



State variables:

- excitation,  $e: \dot{e} = (u - e)/\tau_{ne}$
- activation,  $a: \dot{a} = (e - a)/\tau_a$
- length contractile element,  $l_{ce}$



Based on [Winters and Stark \(1985\)](#)

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 29 of 42

Go Back

Full Screen

Close

Quit

### 5.3. Inverse dynamics summary

So far we have seen:

- model input: closed chain mechanism
- optimisation: efficiency, cost function
- GH stability, scapulo-thoracic contact
- examples: joint forces, moment balance
- muscle dynamics

But, we would really like forward dynamics...

*Introduction*

*Equations of motion*

*Inverse-dynamic...*

*Examples of inverse...*

*Muscle dynamics*

*Forward dynamics*

*Examples of forward...*

*Summary*

*Title Page*



*Page 30 of 42*

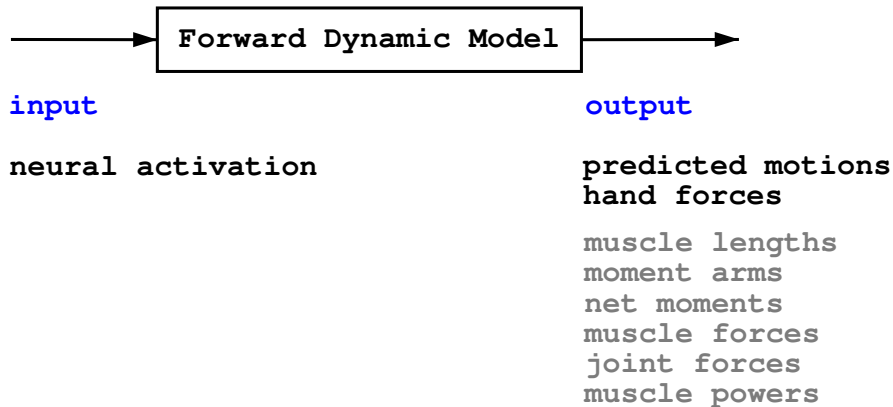
*Go Back*

*Full Screen*

*Close*

*Quit*

## 6. Forward dynamics



*Introduction*

*Equations of motion*

*Inverse-dynamic...*

*Examples of inverse...*

*Muscle dynamics*

***Forward dynamics***

*Examples of forward...*

*Summary*

*Title Page*



*Page 31 of 42*

*Go Back*

*Full Screen*

*Close*

*Quit*

## 6.1. Why do we need this?

Forward-dynamic models have some advantages:

- no *a priori* assumptions about kinematics
- output (motions) easy to visualise and relate to function
- modelling of stiff structures such as ligaments possible
- computer-assisted surgery

But one big disadvantage:

- optimisation requires repeated integration of the system and is thus computationally *very* expensive
  - not suitable for large-scale models

However, we already have a set of optimum inputs (5.1) so no further optimisation is necessary.

These inputs can be used to drive the FD model  $\Rightarrow$

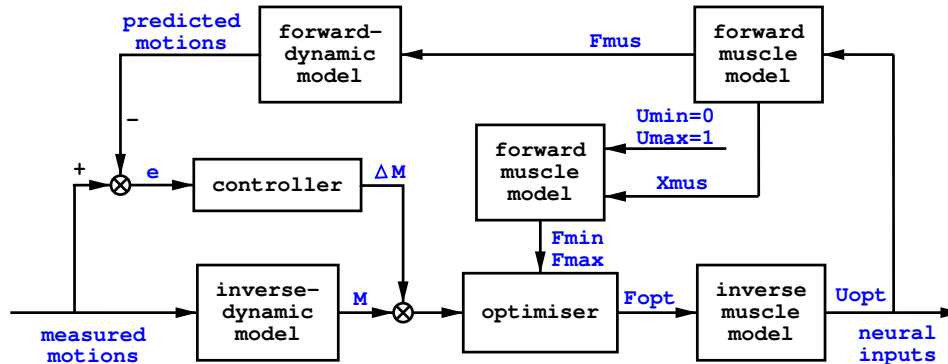
[Title Page](#)


Page 32 of 42

[Go Back](#)
[Full Screen](#)
[Close](#)
[Quit](#)



### 6.3. Inverse-Forward Dynamic Optimisation with Controller



- forward muscle model ensures physiologically feasible solutions for muscle force optimisation
- controller ensures calculated neural inputs reproduce measured motions

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 34 of 42

Go Back

Full Screen

Close

Quit

## 6.4. Integration routines

Euler:

- predictor method, constant step-size
- fast but not very accurate

Adams-Moulton:

- predictor-corrector method, variable step size
- accurate but slow

Runge-Kutta:

- predictor method
- good combination of accuracy and speed

Further reading: [Lennox and Chadwick \(1977\)](#) and [Press et al. \(1992\)](#).  
See also: <http://mathworld.wolfram.com/OrdinaryDifferentialEquation.html>

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 35 of 42

Go Back

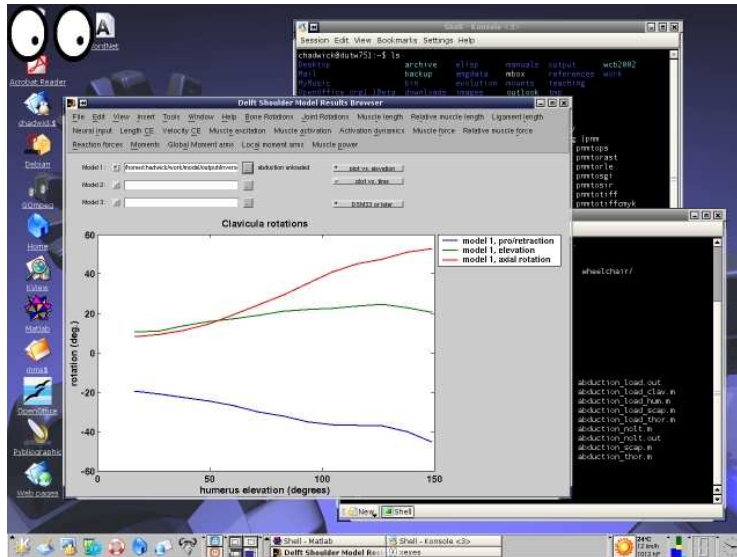
Full Screen

Close

Quit

## 7. Examples of forward dynamics

### 7.1. Matlab results viewer



Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 36 of 42

Go Back

Full Screen

Close

Quit

## 7.2. Prosthesis placement

**Question:** How sensitive is shoulder function after arthroplasty to prosthesis placement?

**Context:** Unknown effect on post-operative motions, neural input known for standard motions (from healthy volunteers).

**Tool:** Forward-dynamic model (sensitivity analysis).

**Application:** Specifications for improved operative technique for prosthesis implantation.

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 37 of 42

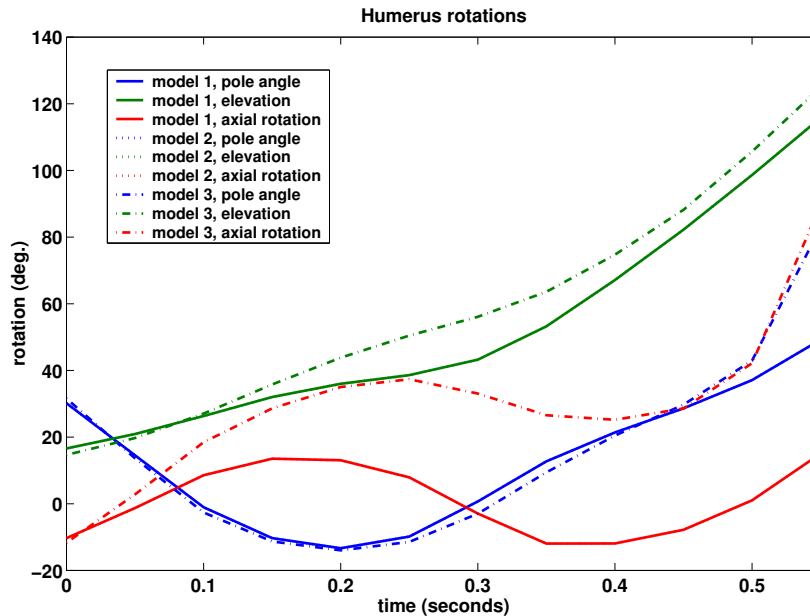
Go Back

Full Screen

Close

Quit

## 7.2.1. Rotations of the humerus



chain line shows rotations of humerus with displaced joint rotation centre (1cm laterally)

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 38 of 42

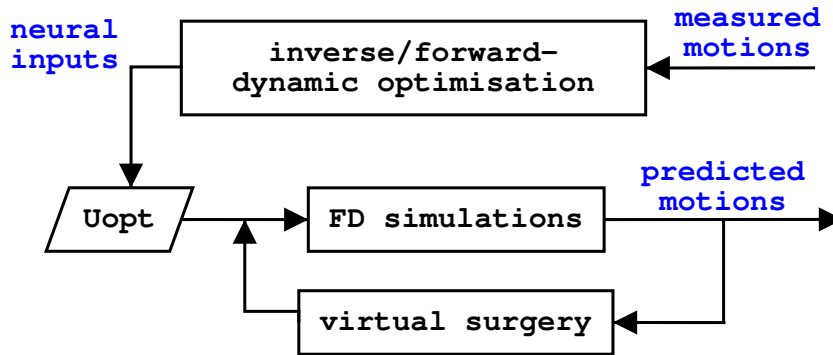
Go Back

Full Screen

Close

Quit

### 7.3. Computer-assisted surgery



1. Run IFDOC to get neural inputs
  - using healthy motions
2. Carry out CAS using only FD model
  - no optimisation necessary → fast simulations

## 8. Summary

Things to consider when modelling the shoulder:

- 17 DoF, closed-chain mechanism (1.3, 3.2)
- Equations of motion: TMT method recommended (2.3)
- Load-sharing problem (3.4)
- Constraints: GH stability, gliding plane (3.5)
- Muscle dynamics in optimisation (5)
- Advantages of forward dynamics (6.1)
- Calculation of neural inputs for forward dynamics (6.3)
- But consider application when choosing model (4.6)

This document is also available online at:  
<http://www.internationalshouldergroup.org/>

Introduction

Equations of motion

Inverse-dynamic...

Examples of inverse...

Muscle dynamics

Forward dynamics

Examples of forward...

Summary

Title Page



Page 40 of 42

Go Back

Full Screen

Close

Quit

## References

- Happee, R., Van der Helm, F., 1995. The control of shoulder muscles during goal directed movements, an inverse dynamic analysis. *J Biomech* 28 (10), 1179–91.
- Jonker, J. B., 1984. Dynamics of spatial mechanisms with flexible links. Tech. Rep. 804, Delft University of Technology, Dept. of Mechanical Engineering, wTHD Nr. 171.
- Lennox, S. C., Chadwick, M., 1977. Mathematics for Engineers and Applied Scientists. Heinemann Educational Books.
- Press, W. H., Flannery, B. P., Teukolsky, S. A., Vetterling, W. T., 1992. Numerical Recipes in FORTRAN: The Art of Scientific Computing. Cambridge University Press, Cambridge, England.
- Tsirakos, D., Baltzopoulos, V., Bartlett, R., 1997. Inverse optimization: functional and physiological considerations related to the force-sharing problem. *Crit Rev Biomed Eng* 25 (4-5), 371–407.
- van der Helm, F., 1994. A finite element musculoskeletal model of the shoulder mechanism. *J Biomech* 27 (5), 551–69.
- van Soest, A., Schwab, A., Bobbert, M., van Ingen Schenau, G., 1992. SPACAR: a software subroutine package for simulation of the behavior of biomechanical systems. *J Biomech* 25 (10), 1219–26.

*Introduction*

*Equations of motion*

*Inverse-dynamic...*

*Examples of inverse...*

*Muscle dynamics*

*Forward dynamics*

*Examples of forward...*

*Summary*

*Title Page*



*Page 41 of 42*

*Go Back*

*Full Screen*

*Close*

*Quit*

- Veeger, H., Rozendaal, L., van der Helm, F., 2002. Load on the shoulder in low intensity wheelchair propulsion. Clin Biomech (Bristol, Avon) 17 (3), 211–8.
- Winters, J., Stark, L., 1985. Analysis of fundamental human movement patterns through the use of in-depth antagonistic muscle models. IEEE Trans Biomed Eng 32 (10), 826–39.
- Zajac, F., Neptune, R., Kautz, S., 2002. Biomechanics and muscle coordination of human walking. Part I: introduction to concepts, power transfer, dynamics and simulations. Gait Posture 16 (3), 215–32.

[Introduction](#)[Equations of motion](#)[Inverse-dynamic...](#)[Examples of inverse...](#)[Muscle dynamics](#)[Forward dynamics](#)[Examples of forward...](#)[Summary](#)[Title Page](#)[Page 42 of 42](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)